Eur. Phys. J. B **38**, 387–391 (2004)
DOI: 10.1140/epjb/e2004-00132-y

The European Physical Journal B

Statistical features of drainage basins in mars channel networks

Can one guess from the landscape the past presence of water?

G. Caldarelli[1,4], P. De Los Rios[2], M. Montuori[3,4], and V.D.P. Servedio[1,a]

[1] INFM UdR di Roma1 and Dip. di Fisica, Università "La Sapienza", P.le A. Moro 2, 00185 Roma, Italy
[2] Institut de Physique Théorique, Université de Lausanne, 1015 Lausanne, Switzerland
[3] SMC INFM, Università "La Sapienza", P.le A. Moro 2, 00185 Roma, Italy
[4] Centro Studi e Ricerche E. Fermi, Compendio Viminale, Roma, Italy

Received 6 October 2003
Published online 14 May 2004 –

Abstract. Erosion by flowing water is one of the major forces shaping the surface of Earth. Studies in the last decade have shown, in particular, that the drainage region of rivers, where water is collected, exhibits scale invariant features characterized by exponents that are the same for rivers around the world. Here we show that from the data obtained by the MOLA altimeter of the Mars Global Surveyor one can perform the same analysis for mountain sides on Mars. We then show that in some regions fluid erosion might have played a role in the present martian landscape.

PACS. 92.40.Fb Rivers, runoff, and streamflow – 96.30.Gc Mars – 05.45.Df Fractals

1 Introduction

Whether water flowed on the surface of Mars in the past is one of the most intriguing and challenging problems of contemporary planetary science. Indeed the presence of surface water, even some billion years ago, would make the odds for the development of life (as we know it) on Mars much higher. At present, the atmospheric conditions are almost incompatible with surface liquid water, so that its presence in the past has to be detected indirectly, looking at clues in the geomorphology of the planet. Moreover, understanding where water could have been present in the past is of great importance to choose the landing sites of future automatic and manned missions to Mars. In order to try to solve such problem we propose to adopt an application of a particular set of graphs namely the Drainage Network Basin, i.e. a spanning tree of the planet surface area.

Digital Elevation Maps (DEM) of the Earth surface are the tool that hydrogeologists use to draw the Drainage Network Basin of rivers. Essentially, the DEM gives a pixel representation of a surface, telling the elevation of every pixel. From that representation the standard method to infer the drainage network of a river is to draw a link between every pixel and the nearest neighbor ones according the steepest descent direction. The final result of this procedure (that implies some more details, such as the formation of lakes, see below) is that the surface under investigation is covered by a spanning tree (a network without loops that passes by all pixels of the map). The main observation is that the fractal properties of the network are peculiar of rivers, and different from those of random spanning trees. As such, their fractal properties can be used to discriminate between simply rough regions and regions that have been sculpted, in time, by the erosion of flowing material.

Recently the National Aeronautics and Space Administration (NASA) has published DEM of the surface of Mars. By direct photographic inspection it is possible to identify regions that are similar to terrestrial river channels and networks. We use a quantitative analysis to check whether the similarity goes beyond the pictorial level to also match the same fractal features, and to validate the idea that such regions have indeed been sculpted by the flow of some material (that, given the current knowledge and evidence, has been likely water).

2 The Mars surface data

Already early pictures taken from orbit showed canyons and channels similar to the ones carved by rivers on Earth. More recently, the high resolution pictures taken in the last three years by the Mars Orbiter Camera (MOC) camera of the NASA probe Mars Global Surveyor (MGS) have provided a wealth of data that add some more clues: gullies on the walls of craters and valleys [1], that suggest the presence of liquid water in (geologically) recent times, sedimentary rock formations typical, on Earth, of the beds

[a] e-mail: vito.servedio@roma1.infn.it

of ancient lakes [2], a crustal dichotomy between northern and southern hemisphere which has been interpreted as due to an ocean in the northern terrains [3] and rootless cones similar to the ones that are found on Earth where molten lava has flowed over waterlogged ground [4]. All these photographic evidence needs to be complemented by some more robust analysis on data such as the ones from the Mars Orbiter Laser Altimeter (MOLA). Here we show that the channel networks that can be inferred from the MOLA data bear similarities with the structures carved by water erosion on Earth. A preliminary analysis was first reported in [5] and lately in [6].

3 Fractal river theory

On Earth fluvial systems can be described through the self-affine properties of their drainage basin, where water, either from underground sources or from rain, is collected [7,8]. Those self-affine properties have been established starting from Hack's seminal work where the following scaling relation between the basin area A and the length $L_\parallel$ of the basin's main stream was proposed [9]:

$$A^h \sim L_\parallel. \tag{1}$$

From measures on real rivers, the Hack's exponent h takes values between 0.5 and 0.7. The allometric relation between A and $L_\parallel$ is just one in a number of scaling properties of drainage basins. In general, the drainage basin is characterized by the longitudinal length $L_\parallel$ (we use the same symbol as for the main stream length since, for real rivers, they scale in the same way) and by the transverse one, $L_\perp$ in such a way that $L_\perp \sim L_\parallel^H$. If the exponent H is less than 1, the basin is self-affine (the width of the basin scales slower than its length). If the basin is self-similar H is equal to 1 (the trivial case where the width scales as the length). The two exponents H and h are connected, as it is easy to see by comparing $L_\perp \sim L_\parallel^H$ together with Hack's law (1): one finds that $h = 1/(1 + H)$. These two macroscopic features are not the only ones available in the study of drainage basins. In order to describe even more precisely the statistical properties channel networks, geomorphologists use Digital Elevation Models (DEM) [10,11]. Measures, either ground based or, in more recent times, by satellites, provide the average height of an area that, on Earth, can be as little as 30×30 m^2. Each of these square units is associated to a pixel on an image (or, technically, to a site on a two dimensional square grid). Then, water collected in each pixel flows toward the lowest of the neighbor pixels, according to a simple maximal slope rule (other rules would be also viable, such as, for example, that water flows toward more than a single neighbor, and is partitioned proportionally to the slopes). If the maximal slope rule is applied, this procedure produces a branched structure without loops that should reproduce the visible river network. In a loopless branched structure, it is easy to define the region upstream of a given point, that is, the region whose collected water will all flow to that point. Therefore it is possible to label each and every point on the map by the area A of its own drainage basin. The final outlet is of course labeled by the whole area of the basin. It is possible to draw the histogram of these areas for a given basin, finding that the frequency $P(A)$ to find a sub-basin of area A follows the law $P(A) \sim A^{-\tau}$, where $\tau = 1.43 - 1.46$. On the same branched structure, it is possible to measure the upstream length L from a given point, as the distance of that point from the furthest source (a source is defined as a point without any upstream basin, that is, that does not collect any upstream water). Again, these lengths can be organized in a histogram, whose asymptotic behavior for the frequency distribution $\Pi(L)$ behaves as $\Pi(L) \sim L^{-\gamma}$, where $\gamma = 1.7{-}1.8$. All these exponents, and others that could be defined, depend on the above exponent H relating $L_\perp$ and $L_\parallel$, and then depend ultimately on the fractal properties of the basin. Using finite size scaling arguments, Maritan et al. [12] have shown that $\tau = 2-h$, and $\gamma = 1/h$. These relations hold for any kind of branched structure, whatever its origin, and they show that indeed there is only one independent exponent that determines all the others. Actually, one more independent exponent is the fractal dimension of the main stream; yet, river networks are, on the average, directed because water flows down slopes, which gives a preferential direction. Consequently the fractal dimension of the main stream is $d_l = 1$. Still, the relations do not tell why the exponents should take a particular set of values. The case of multiple outlets is particular in that one can expect slightly different results [13].

Table 1. Exponents for Earth river networks, for Mars channel networks (processed and unprocessed), for the martian northern hemisphere (Exp.=exponential; n.d.=not defined), and for a random surface (processed and unprocessed); in brackets the overlap is reported.

Surface	H	τ	γ
Earth	$0.7 - 0.8$	$1.43 - 1.46$	$1.7 - 1.8$
Mars (90%)	∼0.6	∼1.5	∼2.0
Mars North. Hemisphere	n.d.	Exp.	Exp.
Random Surf.	n.d.	Exp.	Exp.
Random Surf. (20−25%)	1	11/8	5/4

One can study the same statistical properties on a random landscape. In this case of course, due to the nature of the landscape, many outlets (points lower than all of their neighbors) will be present in the system. We will refer hereafter to these points as "pits". These points act as sinks where water is only collected and not distributed around. This effect is so strong, that aggregation of rivers does not take place. Drainage basins have a characteristic small size and the above distributions $P(A)$ and $\Pi(L)$ show an exponential decay. In order to remove inner pits, one can recursively raise them at the height of the lowest neighbor (therefore simulating the behavior of water in a real lake when the liquid level increases steadily). By this procedure, one finally deals with connected structures (maybe with multiple outlets on the boundaries) whose landscape is very different from the original one.

The statistical features of the resulting spanning trees are listed in Table 1. This class of spanning trees has been

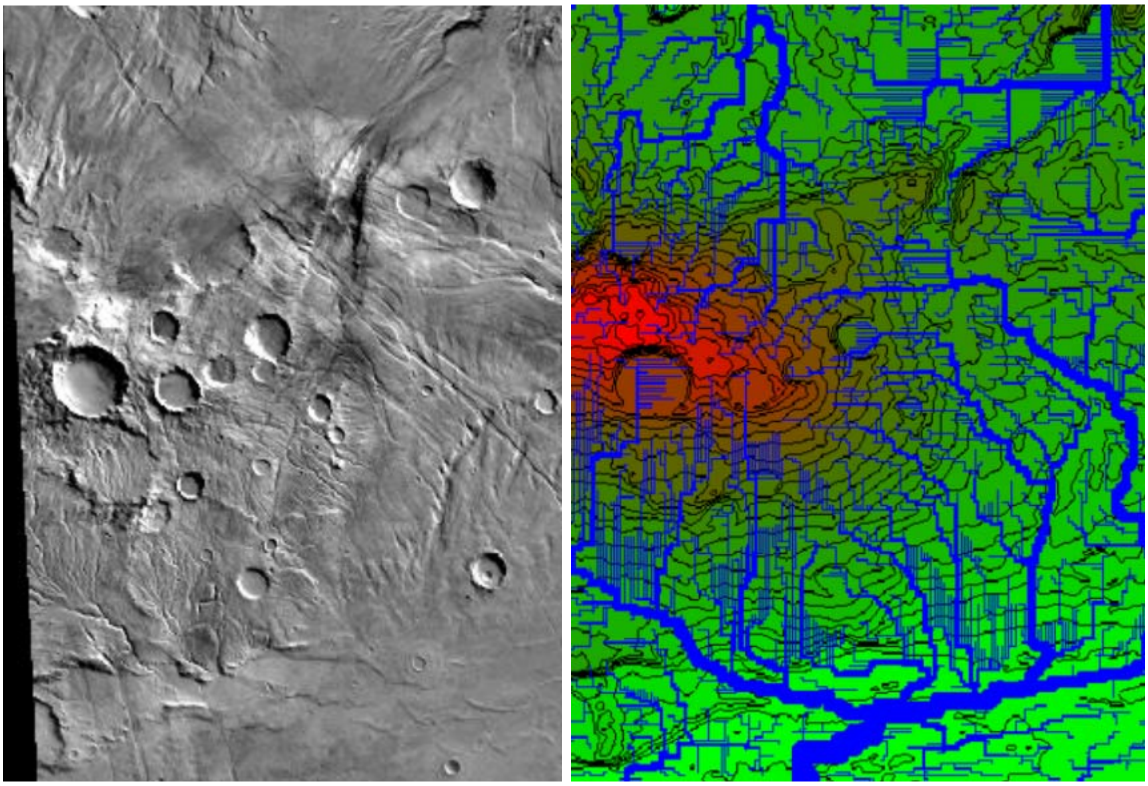

Fig. 1. (left):Viking Image for the Warrego Vallis (around 41°S, 91°W); some "river-like" structures are present on the center-left (courtesy from NASA, PDS Planetary Image Atlas). (right): Computer reconstruction of the landscape in the area of Warrego Vallis, after pit removal. The reconstructed river structure is similar to the visible one on the photograph.

also studied analytically and will be hereafter described as the class of the "random spanning trees" [14]. Since real rivers are not described by this class of spanning trees, it has been conjectured that an optimization process took place in order to shape the drainage basins in their present form. In particular with the aid of the Optimal Channel Networks (OCN) [15] model one can show in a rigorous way that by requiring minimization of the total gravitational energy dissipation in the system one can reshape a random spanning tree, transforming it into one with the same statistical properties of the real basins. We then want to exploit this difference in order to check if the martian landscape has ever been sculpted by fluvial erosion.

4 Data analysis

We performed the same analysis on the surface of Mars. The latest MOLA data provide us with a DEM such that every pixel covers a surface of about 1690×1690 m^2 (roughly $0.03125° \times 0.03125°$). Even if data with a larger resolution have not been released yet, we have been able through MOLA software to extract information on a grid where any pixel covers an area of 845×845 m^2. With these data at hand, we have chosen four regions of about 100×100 km^2, where to reconstruct a network from a DEM: the *Warrego Vallis* (W.V.) shown in Figure 1 (left), where some river-like structures are visible in both Viking Orbiter and MOC pictures; *Solis Planum* (S.P.) and *Noctis Labyrinthum* (N.L.), two rough districts in the Tharsis region; and a region from the northern hemisphere, where an ocean could have existed in the past [3]. On these regions we reconstructed a channel network using the maximal slope rule: the results are shown in Figure 1 (right), where we represented every pixel by the area it collects (at this level we still do not speculate whether such an area corresponds to any collected water). In Figure 2 we show the statistical analysis of the areas for the Warrego Vallis: we find indeed that $P(A)$ before removing pits seemingly exhibits a power-law decay with exponent τ close to 1.5 for small areas, and between 1.7 and 1.9 for large ones. The same result holds for the other terrains, Solis Planum and Noctis Labyrinthum (not shown), whereas the norther hemisphere areas decrease exponentially (shown in Fig. 2). These first results suggest that the northern lowlands are extremely flat, without any long-range correlated structure. On the contrary, the three other regions show some correlated structures, at least for small areas. As it is shown in the inset, where binned data are reported (they are binned on powers of 1.5), asymptotically the area distribution before pit removal shows a clear cutoff. To check whether the cutoff is a consequence of pits, we resort to the pit removal procedure. The corresponding network for the Warrego Vallis is shown in Figure 1 (right). Now, long

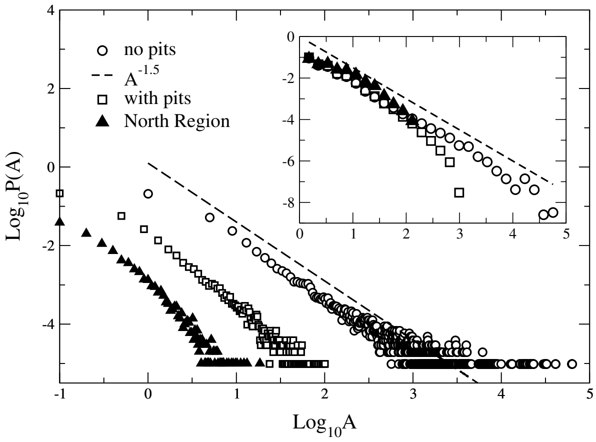

Fig. 2. Histogram of the areas for the Warrego Vallis and for the northern lowlands. Data before pit removal and for the northern lowlands have been shifted for convenience. In the inset, the same data are shown binned on powers of 1.5 to average away some noise.

rivers are present, and the new statistics agree with the preceding ones for small areas, and increase the cutoff so that the exponent 1.5 holds also for intermediate to large areas. The exponent γ cannot be fitted before pit removal, whereas it is close to 2 after pit removal, consistent with the relation $\tau = 2 - 1/\gamma$. We stress here that the exponent $\tau = 1.5$ cannot be simply the result for a network reconstructed on a random slope. Both numerically and analytically it has been found that in this case $\tau = 4/3$ [16]. Therefore the exponent $\tau = 1.5$ is different from both flat and sloped random networks.

5 Conclusions

As mentioned above, pits are removed from the landscape with the standard procedure of DEM on Earth. One lets the water fill them up to the level of the lowest neighbor, and then flow toward it. Since the simplicity of such procedure is given by the physics of gravity, there is no reason to believe it could not be applied even on Mars. If this procedure is applied then the only possible outlets are located on the boundaries. We can find an a posteriori measure of the effect of this procedure. Indeed if the surface is a random one, we can expect a large proportion of pits to be removed, and the network should greatly change. If instead feedback between fluvial erosion and landscape evolution took place in the past, one should expect a very little proportion of removed pits and few flow directions should change. Therefore a simple criterion can be used to decide how much this procedure changes the network: we define the overlap between the processed and unprocessed networks as the fraction of sites that do not change their outflow direction. A high value of the overlap means that filling pits does not change much the network, whereas a low overlap implies that the network has been completely modified. Using this criterion, we can see that a random network changes almost completely upon processing, whereas the three channel networks (W.V., S.P. and N.L.) have large overlaps ($\geq$85%), implying that no large rearrangements have taken place, and that therefore it is tempting to associate such networks to real rivers. In the case of W.V. this overlap becomes greater then 94% for the upper drainage basin. For a random surface, the pits removal will produce a different landscape, which is expected to have an overlap of 25% with respect to the original one. The much larger value we got for the martian surface under analysis is an evidence of the little effect of pits removal on the statistical features of the network basin.

Moreover, by considering where rearrangements are located spatially, one can note that they are mainly in the crater areas, thereby they can be put in correlation with geophysical phenomena more recent than fluvial erosions.

In comparison with data available on Earth (30×30 m^2), the resolution available at present is quite low. Yet, we obtained the same results working on maps of 845 m $\times$ 845 m, the highest resolution available at the moment. This analysis then is quite important, since one needs to extract the maximum information available from this data set at the time of the first submission.

A further problem with the interpretation of the MOLA data is that we are trying to look at channel networks produced maybe billions of years ago. Over such a huge period, winds in the thin martian atmosphere could have smoothed the landscape [17]. Going back to the original landscape is quite difficult; erosion rates can be greatly varied by climate changes. Nevertheless, a simple idea could be to apply some anti-diffusion, but the starting landscape has a resolution such that we could not be sure about the outcome (anti-diffusion is an unstable process). Yet, we can try to look at the behavior of an even more eroded landscape, to understand how much erosion influences the area and upstream length statistics. Applying some diffusion (that, on the contrary, is a stable process), we find smoother and smoother surfaces, yet the statistics do not change considerably. If we extrapolate back in the past the same considerations, we can conclude that today we are looking at statistics close to the ones when the networks have been produced.

Answering the question on whether water ever flowed on Mars is of extreme importance: Mars is the closest place that could have harbored life in the past, close enough that, at some point in the near future, we could have direct evidence of it. Moreover, understanding where water was present in the past could also be of relevance in choosing the landing site for future human missions: the site should be as much scientifically interesting as possible. The present study tries indeed to point out the possible scientific interest of some regions. From the landscape properties we are able to show a long range correlation that can be explained only by hypotizing presence of fluvial erosion. Even if the data resolution is quite poor we can distinguish different geophysical properties in different regions of the Mars surface.

We thank the MOLA Team at NASA for making the MOLA data readily available online, and for helping us to extract the information for our analysis. This work has been partially supported by the EU Network ERBFMRXCT980183. M.M. acknowledge support from INFM Forum on clustering.

References

1. M.C. Malin, K.S. Edgett, Science **288**, 2330 (2000)
2. M.C. Malinand K.S. Edgett, Science **290**, 1927 (2000)
3. J.W. Head et al., Science **286**, 2134 (1999)
4. P.D. Lanagan, A.S. McEwen, L.P. Keszthelyi, T. Thordarson, Geophys. Res. Lett. **28**, 2365 (2001)
5. G.Caldarelli, P. De Los Rios, M. Montuori, *Statistical Features of Drainage Basins in Mars Channel Networks*, `http://lanl.arxiv.org/cond-mat/0107228` (2001)
6. T.F. Stepinski, M.M. Marinova, P.J. McGovern, S.M. Clifford, Geophys. Res. Lett. **29**, 30 (2002)
7. I. Rodriguez-Iturbe, A. Rinaldo *Fractal Rivers Basins, Chance and Self-Organization* (Cambridge University Press, Cambridge, 1997)
8. D.G. Tarboton, R.L. Bras, I. Rodriguez-Iturbe, Water Resour. Res. **24**, 1317 (1988)
9. J.T. Hack, U.S. Geol. Survey Prof. Paper 294-B, 1 (1957)
10. L. Band Water Resour. Res. **22**, 15 (1986)
11. W.E. Dietrich, C.J. Wilson, D.R. Montgomery, J. McKean, J. Geol. **20**, 675 (1992)
12. A. Maritan et al., Phys. Rev. E **53**, 1510 (1996)
13. G. Caldarelli, Phys. Rev. E **63**, 21118 (2001)
14. A. Coniglio, Phys. Rev. Lett. **62**, 3054 (1989)
15. I. Rodriguez-Iturbe et al., Water Resour. Res. **28**, 1095 (1992)
16. A.E. Scheidegger, Water Resources Research **6**, 750 (1970)
17. Z. Peizhen, P. Molnar, W.R. Downs, Nature **410**, 891 (2001)